PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES

Second Edition

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Memoryless Systems

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A system is called memoryless if its output is given by

$$y(t) = g[x(t)]$$

where $g(x)$ is a function of x. Thus, at a given time $t = t_1$, the output $y(t_1)$ depends only on $x(t_1)$ and not on any other past or future values of $x(t)$.

From the above it follows that the first-order density $f_y(y; t)$ of $y(t)$ can be expressed in terms of the corresponding density $f_x(x; t)$ of $x(t)$ as in Sec. 5-2. We note, in particular, that

$$E\{y(t)\} = \int_{-\infty}^{\infty} g(x) f_x(x; t)\, dx$$

Similarly, since $y(t_1) = g[x(t_1)]$ and $y(t_2) = g[x(t_2)]$, the second-order density $f_y(y_1, y_2; t_1, t_2)$ of $y(t)$ can be determined in terms of the corresponding density $f_x(x_1, x_2; t_1, t_2)$ of $x(t)$ as in Sec. 6-3. In particular

$$E\{y(t_1)y(t_2)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, g(x_1) g(x_2) f_x(x_1, x_2; t_1, t_2)\, dx_1\, dx_2$$

The nth order density $f_y(y_1, \ldots, y_n; t_1, \ldots, t_n)$ of $y(t)$ can be determined from the corresponding density of $x(t)$ as in (8-8) where the underlying transformation is the system

$$y(t_1) = g[x(t_1)], \ldots, y(t_n) = g[x(t_n)] \qquad (9\text{-}86)$$

Stationarity Suppose that the input to a memoryless system is an SSS process $x(t)$. We shall show that the resulting output $y(t)$ is also SSS.

PROOF To determine the nth order density of $y(t)$, we solve the system

$$g(x_1) = y_1, \ldots, g(x_n) = y_n \qquad (9\text{-}87)$$

If this system has a unique solution, then [see (8-8)]

$$f_y(y_1, \ldots, y_n; t_1, \ldots, t_n) = \frac{f_x(x_1, \ldots, x_n; t_1, \ldots, t_n)}{|g'(x_1) \cdots g'(x_n)|} \qquad (9\text{-}88)$$

From the stationarity of $x(t)$ it follows that the numerator in (9-88) is invariant to a shift of the time origin. And since the denominator does not depend on t, we conclude that the left side does not change if t_i is replaced by $t_i + c$. Hence, $y(t)$ is SSS. We can similarly show that this is true even if (9-87) has more than one solution.

Notes 1. If $x(t)$ is stationary of order N, then $y(t)$ is stationary of order N.
2. If $x(t)$ is stationary in an interval, then $y(t)$ is stationary in the same interval.
3. If $x(t)$ is WSS stationary, then $y(t)$ might not be stationary in any sense.

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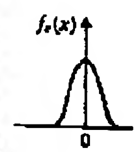

Figure 9-9